NPS-61-83-010

NAVAL POSTGRADUATE SCHOOL

Monterey, California

DIFFRACTION EFFECTS IN
CERENKOV RADIATION

John R. Neighbours and Fred R. Buskirk

June 1983

Technical Report

Approved for public release; distribution unlimited

Prepared for:
Chief of Naval Research
Arlington, Virginia 22217

NAVAL POSTGRADUATE SCHOOL
Monterey, California

Rear Admiral John Ekelund
Superintendent

David Schrady
Provost

The work reported herein was supported by the Chief of Naval Research.

This report was prepared by:

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NPS-61-83-010	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Diffraction Effects in Cerenkov Radiation		5. TYPE OF REPORT & PERIOD COVERED Technical Report
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) John R. Neighbours Fred R. Buskirk		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, CA 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 62768N: RF68-342-800 N000148WR20195
11. CONTROLLING OFFICE NAME AND ADDRESS Chief of Naval Research Arlington, VA 22217		12. REPORT DATE June 1983
		13. NUMBER OF PAGES 29
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Cerenkov radiation
Microwave radiation
Bunched electron
Relativistic electron beams

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Previous work has shown that microwave Cerenkov radiation from periodic electron bunches occurs at harmoics of the bunch frequency and that the radiation is not necessarily confined to a narrow region centered about the expected Cerenkov angle. This report continues the investigation of the spreading of the radiation and presents the results of numberical calculations of the radiated power per unit solid angle and the total radiated power for physically realizable conditions.

DD FORM 1473 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

TABLE OF CONTENTS

DIFFRACTION EFFECTS IN CERENKOV RADIATION

John R. Neighbours and Fred R. Buskirk
Physics Department

Naval Postgraduate School
Monterey, CA. 93940

INTRODUCTION

In previous work[1] Cerenkov radiation was considered for periodic bunches of electrons such as would be emitted by a typical travelling wave electron accelerator (Linac). Even in air, electrons from a 100 MeV Linac exceed the velocity of light in the medium, and Cerenkov radiation should be emitted at a small angle of less that 2 degrees. The radiation intensity was calculated in detail for microwave frequencies and some of the results were as follows:

a. For bunches periodic in time with frequency ν_o, radiation is emitted at ν_o and harmonics therof, in contrast to the continuous frequency distribution observed for a single charge.

b. If each bunch has a spatial distribution described by a charge density $\rho_o'(\vec{r})$, the radiated intensity is modified by the fourier transform of this charge distribution.

c. At low frequencies such that the wave length of the emitted radiation is of the order of the bunch size, the electrons in the bunch radiate coherently. This leads to large enhancement factors for typical linac bunches consisting of 10^8 electrons, and in fact allows the radiation to be significant at microwave frequencies. Destructive interference, described by the fourier transform of the charge density, decreases intensities with increasing frequency, until incoherent radiation takes over when the wave length of the radiation is much less than the electron spacing.

d. If the emission region has finite length, which may be realized by passing the electron beam through a gas cell, the radiation propagation direction is not confined to a sharp Cerenkov angle θ_C, where cos θ_C is defined to be c/v, but is spread over a range of emission angles. This spreading effect depends only on the length of gas cell, and does not depend on the electron bunch structure or periodicity of the bunches. In this paper as well as 1, c is the speed of light in the medium.

The spreading of the emission angle noted in (d) above, as calculated in the earlier paper, contained approximations which are invalid in many common situations, so that the situation is considered in more detail here.

GENERAL AND LIMITING RESULTS

The power radiated from periodic bunches of electrons traveling at a velocity v greater that the speed of light c in a medium has been given in Equations 25, 26 and 27 of reference 1. For convenience they are written below using in slightly different terminology. The power radiated per unit solid angle $dp/d\Omega$ is

$$\frac{dP}{d\Omega} = r^2 \frac{1}{T} \int_0^T \hat{n} \cdot \vec{S}\, dt = \frac{2r^2}{\mu} \sum_0^\infty \frac{\omega^2}{C} \left| \hat{n} \times \underset{\sim}{\vec{A}}(\vec{r},\omega) \right|^2 \tag{1}$$
$$= \sum_0^\infty W(\nu,\hat{n})$$

where $W(\nu,\hat{n})$, the power per unit solid angle radiated at the frequency ν, was shown to be

$$W(\nu,\hat{n}) = \frac{\mu}{2c} L^2 \nu^2 \nu_o{}^2 \sin^2\theta \left| \underset{\sim}{\rho}{}_o'(\vec{k}) \right|^2 I^2(u) \tag{2}$$

The parameters describing the radiation are

$$u = \frac{kL}{2} (\cos\theta_c - \cos\theta)$$
$$I(u) = \frac{\sin u}{u} \tag{3}$$
$$\vec{k} = (n_x \frac{\omega}{c}, n_y \frac{\omega}{c}, \frac{\omega}{v})$$

$$\underset{\sim}{\rho}{}_o'(\vec{k}) = \iiint_{-\infty}^{\infty} d^3r\, e^{-i\vec{k}\cdot\vec{r}} \rho_o'(\vec{r})$$

where n_x, n_y, n_z are components of the unit vector $\hat{n}$ in the emission direction, ν is the frequency of the emitted radiation, $L(=2Z')$ is the length of the gas cell, $\rho_o'(\vec{r})$ is the charge density distribution for one bunch, and the usual Cerenkov angle θ_c is given by $\cos\theta_c = {}^c/v$. The bunch frequency ν_o is equal to the electron velocity divided by Z, the electron bunch spacing ($\nu_o = v/Z$) and ν the frequency of the emitted radiation will be a harmonic of ν_o.

The results above are general ones for the emission of radiation from periodic bunches of electrons. In (2) the factor $L^2 I(u)^2$ is identical to the result obtained for the calculation of elementary Fraunhofer diffraction from a slit across which the phase varies linearly such as plane waves striking the slit at an angle to the normal. However in (2) the diffraction angle is not the usual one measured from the normal to the radiating line source. Here it is more convenient to measure the diffraction angle θ from the electron beam line so that it is the complement of the usual one.

The $\sin^2\theta$ factor arises from cross products used in calculating the Poynting vector. It is just the angular factor in the usual expression for the power radiated from a dipole oriented along the electron beam. Thus the expression (2) for W can be interpreted as the interference of dipole radiators whose phase varies linearly with position along the beam line.

In the expression (2) for the radiated power per unit solid angle, the factor $I(u)^2$ is strongly peaked at u = o. (When u = o, the radiation angle is equal to the Cerenkov angle θ_c.)

In an earlier paper W was integrated over solid angle to obtain the total radiated power, assuming that I was so strongly peaked that the other functions in the integrand, namely $\sin^2\theta$ and $\underset{\sim}{\rho}'_o(\vec{k})$, could be evaluated at $\theta = \theta_c$ and taken outside the integral. Fig. 1 of the previous paper shows the behavior of I^2 as a function of θ, and there is defined a width $\Gamma = \Delta G = \pi/2Z'$. If, in fact, the $\sin\theta$ and $\underset{\sim}{\rho}'_o(\vec{k})$ factors are slowly changing through the peak in I, the radiation is concentrated near the Cerenkov angle θ_c and the total radiated power is given by Eq. (32) of the earlier paper, which is

$$P_{\omega'} = \frac{\mu}{4\pi}\,\omega\omega_o\, v\,\sin^2\theta_c\,|\underset{\sim}{\rho}'_o(\vec{k})|^2 N \tag{4}$$

where N is the number of pulses in the interacting region.

When I(u) is not so strongly peaked, then the variation of the other terms $\sin^2\theta$ and $\underset{\sim}{\rho}'_o(\vec{k})$ in the expression for W must be considered. In general both factors should effect the position and height of the maximum in W with the $\sin^2\theta$ term being predominant at smaller angles.

It is convenient to obtain a more specialized expression for W, and to that end, to introduce the harmonic number $j = \omega/\omega_o$ and to assume particular a charge density distribution,

$$\underset{\sim}{\rho}'_o(\vec{k}) = q\,\mathrm{Exp}\left(-\frac{b^2k^2}{4}\cos^2\theta - \frac{a^2k^2}{4}\sin^2\theta\right) \tag{5}$$

which corresponds to a spatial charge distribution defined by a gaussian

$$\rho_o'(\vec{r}) = C_1 \, \mathrm{Exp}\left(-\frac{x^2}{a^2} - \frac{y^2}{a^2} - \frac{z^2}{b^2}\right) \tag{6}$$

where C_1 is a normalization constant related to the total charge per bunch and a, b are size parameters for the radial and longitudinal dimensions of the bunch.

Then (2) becomes

$$W(\nu,\theta) = \frac{\mu\nu_o^4}{2c} q^2 L^2 \times \left\{ j^2 I^2(u) \sin^2\theta \, \mathrm{Exp}\left(-\frac{b^2k^2}{2} \cos^2\theta - \frac{a^2k^2}{2} \sin^2\theta\right)\right\} \tag{7}$$

For a narrow electron beam, a, the parameter describing the radial dimension of the bunch can be neglected in (7), so that W can be written

$$W(\nu,\theta) = W_j(\theta) = Q \, D_j(\theta) \tag{8}$$

where the radiation function is

$$D_j(\theta) = \frac{j^2}{4} L^2 \sin^2\theta \left(\frac{\sin u}{u}\right)^2 \mathrm{Exp}\left(\frac{-k_z^2 b^2}{2}\right) \tag{9}$$

the constant Q is

$$Q = \frac{2\mu\nu_o^4 q^2}{c}$$

and the diffraction variable is

$$u = \frac{kL}{2} (\cos\theta_c - \cos\theta) \tag{10}$$

and k_z has been written for $k \cos\theta$.

It is worthwhile noting that the frequency of radiation enters into W not only as the $j^{\underline{th}}$ harmonic of the fundamental frequency ν_o but also through the wave vector $\vec{k}$ (in the medium) in the expression (10) for the diffraction variable.

MICROWAVE CERENKOV RADIATION IN A GAS

Equation (5) is generally applicable for gaussian electron bunches moving in a dielectric at velocities greater than the speed of light and (8) is the special case for a fine beam. The central result of this work is a detailed discussion of the results predicted by (8) for the radiated microwave power in a gaseous medium.

The Cerenkov angle defined by $\cos\theta_c = c/v = 1/n\beta$ is small for most gases because n, the index of refraction of the gas is very close to one. Consequently the dependance of θ_c on the electron velocity is very slight since β must also be close to one in order to attain the Cerenkov condition. Thus even for extremely relativistic electrons traveling in a typical gas, θ_c is always small (less than 2 degrees).

In many situations, the factor $(\sin u)^2/u^2$ appearing in (8) and (9) will be essentially a delta function of u, and the radiation will appear at or very near the Cerenkov angle θ_c. This limiting behavior occurs as $kL \to \infty$, with a result given by (4).

Inspection of (10) shows that $u = o$ when $\theta = \theta_c$; a condition which leads to a maximum in the diffraction function $I^2(u) = (\sin u/u)^2$. Further inspection shows that if kL is large, then u will be large at angles significantly different from θ_c, leading to $I^2(u)$ varying rapidly as a function of θ. In the limit as $kL \to \infty$ the diffraction function behaves like a delta function, having only the central maximum at $u = o$. This limiting behaviour occurs regardless of the particular value of θ_c.

For finite kL, the radiation is emitted with a finite range of angles centered about an angle other than θ_c. Although the proper variable for discussing the radiation pattern is the product kL, it more convenient to discuss predicted results in terms of the variation with either k alone or only L alone.

In the range of θ in the vicinity of θ_c, the $\sin^2\theta$ factor in (8) changes very rapidly although $\underset{\sim}{\rho}'_o(\vec{k})$ changes very little. Consequently the radiation then becomes not only smeared about the Cerenkov angle, but the angles greater than θ_c account for more radiated power than the angles between 0 and θ_c. The result is that not only is the radiation distributed in angle about θ_c, but the distribution is distorted so the the peak of intensity may occur at several times θ_c.

To calculate and plot expected results a set of parameters were chosen close to those available in an experiment. A dielectric constant of $\varepsilon = 1.000536$ was assumed for air giving an index of refraction of $n = 1.000268$. The electron parameters were assumed to be those for a 100 MeV linac: fundamental frequency $\nu_o = 2.85$ GHz and a gaussian bunch parameter of $b = 0.24$ cm. The gas cell length was assumed to be 90 cm - also experimentally attainable.

Under these conditions, the calculated behavior of $D_j(\theta)$ is shown in Fig. 1 for the two harmonics $j = 3$ and $j = 5$. The expected Cerenkov angle is calculated to be $\theta_c = 1.29°$ and the striking result is how far the maxima in the high intensity (first) lobes are displaced from θ_c. Each harmonic is displaced differently with the lower one displaced further from θ_c. In an earlier paper the radiated power was evaluated assuming the $\sin^2\theta$ factor to be constant; but even in that approximation it was noted that higher harmonics would be spread less from θ_c.

Further consideration of Fig. 1 also suggests the large amount of total power radiated into larger angles, this latter effect being enhanced by an additional $\sin\theta$ factor (from the solid angle) which multiplies (8) in the expression for total power. To illustrate, also, that the spreading of intensity about the Cerenkov angle is an interference effect associated with the finite gas cell length, $D_j(\theta)$ is calculated for several cell lengths, 70, 90 and 150 cm (usually 90 cm is assumed elsewhere in this paper) for the harmonic j = 3 in Fig. 2. For longer cells, the peak in the main lobe moves toward θ_c but the approach is very slow. However, it is understood rather easily: the first null occurs when the Huygens waves emitted from the front and rear of the cell (of length L) differ in phase by 2π, which gives simply the condition (see appendix B)

$$\frac{L\omega}{c} \cdot (\cos\theta_c - \cos\theta) = 2\pi \tag{11}$$

Then the reason for the large breadth of the lobe is that for the assumed condition, $\cos\theta$ varies slowly so that a large change in θ is required for the 2π phase shift required by (11) for a null.

To show what might be observed in an experiment, Fig 3 displays the sum of $D_j(\theta)$ for harmonics j = 3, 4, 5, and shows the expected washing out of diffraction zeros which occur at different emission angles for different harmonics. These harmonics would be seen by an X-band detector, with sensitivity from 8 to 12 Ghz, assuming ν_o = 2.85 Ghz for an S-band Linac. The effect of adding the three radiation functions is to enhance and shift the first lobe, and to smear out the interference effects at higher angles since the individual nulls in the D_j occur at different angles.

As the product kL increases the position of the maximum in the radiation pattern decreases smoothly to approach the Cerenkov angle. This behaviour is shown in Fig 4 where L is increased keeping the frequency (and thus k) fixed at the value for the third harmonic. The approach to θ_c is smooth but significant differences are to be expected even at very long cell lengths of 100 m.

POWER RADIATED

In the earlier sections it was shown that power is radiated in a range about the Cerenkov angle, as is shown in Fig. 1 and Fig. 2. It is interesting to compare the total power radiated with the predictions of the approximation used in the earlier paper. In Fig. 5, two measures of power radiated are plotted as functions of frequency: (A) the total power and (B) the power integrated over the main diffraction lobe. Also shown is the peak power per unit solid angle in the main lobe. These display a rise at low frequency characteristic of Cerenkov radiation followed by a fall off at high frequencies, associated with the form factor of the bunch. The total power has a shape similar to that merely sketched in Reference 1. The peaks in (A) and (B) occur at different frequencies and the approximate expression (4) (Eq. 32 of ref. 1) would peak at still a different frequency.

A most surprising result is obtained when the total power and the power in the main lobe are compared to the approximate value. These results are given in Table I; additionally, the ratio of the total power to that predicted by the approximation is tabulated in the last column. This ratio has values of about 50 for the lower harmonics, and gradually falls off at the higher frequencies. Noting from Fig. 5 that the power spectrum peaks at about the third or fourth harmonic, the above ratio has values from 35 to 43, meaning that the total Cerenkov radiation per unit length, under assumed conditions would be larger by a factor of about 30 compared to what is expected from an infinite gas cell.

The physical reason for this large increase in the power radiated from a finite cell or path length may be illuminated by examining (2). The factor $\rho_o'(\vec{k}) = q\, F(\vec{k})$ describes the distribution of charge in the bunch and merely allows coherence at low frequencies, but partial cancellation as the wave length of emitted radiation approaches the bunch length, and does not contribute to the power increase. The latter effect is governed by the two factors $\sin^2\theta$ and $I^2(u)$. $I^2(u)$ through its argument u governs phase matching of the electron and the emitted wave from the beginning to the end of the gas cell (of length L'). Perfect phase matching occurs for $u = 0$ or $\cos\theta = \cos\theta_c = c/v$. But for a finite cell, perfect phase matching is not required; as θ changes from θ_c, the radiation from the various parts of the cell tends to cancel. In particular, a null should occur when the radiation from the front and rear elements are out of phase by 2π. This phase difference of 2π is obtained at an angle θ_N for which $kL = 2\pi$, (see appendix 1) and $I(u) = 0$. The fact that u contains $\cos\theta$ and θ is near zero for a gas results in a relatively broad peak. This breadth, combined with the $\sin^2\theta$ factor in the intensity results in the large intensity emitted at angles larger than θ_c, between θ_c and the angle for the first null, θ_N. Also the $\sin^2\theta$ factor accomplishes the increase in power, even if θ_c is small enough so that the null for θ less than θ_c (i.e. $2u \;\; = -2\pi$) is out of the physical range $\cos\theta \leq 1$.

CONCLUSIONS

The object of this paper is to consider in more detail the phenomenon predicted earlier, that the Cerenkov radiation, in cases where the interaction region is finite, should be broadened about the Cerenkov cone. For the case of a gas, for which the Cerenkov angle is a few degrees, the broadening is asymmetric about the Cerenkov cone, and the radiated power displays various interference lobes. Considerable power (about half in the case of a gas) appears outside the main lobe, and the latter is peaked at angles much larger than the Cerenkov angle.

An even more interesting result is that the Cerenkov power radiated per unit path length is increased by large factors approaching two orders of magnitude for a gas cell of finite length compared to one of infinite length.

This increase in power is associated with the finite length of the radiating medium only. The effect should be approximately the same for either single or periodic electron bunches, and should occur whether the bunches are effectively point charges or have significant size.

To recall results obtained earlier, periodic bunches produce Cerenkov radiation at harmonics of the bunch frequency, while a single bunch will radiate with a continuous frequency distribution. The effect of the bunch size is reflected in the factor of the fourier transform of the charge in the bunch, which causes the radiation to fall off at high frequencies while at low frequencies all charges in the bunch radiate in phase.

The results outlined previously might occur in other situations or applications. The distinct phenomena are: (A) Coherent radiation by an electron bunch for wavelengths larger than the bunch, (B) Radiation at harmonics of the bunch frequency, (C) Smearing of the Cerenkov angle and (D) Asymmetric smearing of the Cerenkov angle with an increase in the power radiated. C and D should occur for finite radiator length; D should occur only if θ_C is very small.

All of the above are easily realized in the microwave region, where appropriate bunching occurs for travelling wave accelerators. Possibly useful sub-millimeter wavelength radiation could be realized.

In the optical range, C and D could occur for radiating cells which could be as large as 100 μ meter in length. A, B, C and D together could only be realized with small scale bunching that occurs in a free electron laser.

For the X-ray region it is impossible to envision bunching fine enough to accomplish A and B. The dielectric constant is less than unity, so that Cerenkov radiation does not generally occur. It might be possible to accomplish C and D if small spectral regions could be found in a medium for which the Cerenkov condition is satisfied such as near an atomic resonance[3]. Then greatly enhanced radiation could be produced by using a thin layered structure for a medium.

Table I

Total radiated power as determined by numerical integration and by the approximation of reference 1. Power in the first lobe of the radiation patttern is also included. Parameters are as described in the text. Units are arbitrary.

Harmonic	A Total Power (Numerical Integration)	B Power in 1st Lobe (Numerical Intergration)	C Total Power (Approximation)	A/C
n = 1	11.86	6.65	.1174	101.02
2	13.53	6.59	.228	59.34
3	14.12	6.34	.324	43.58
4	14.12	5.96	.402	35.12
5	13.71	5.49	.459	29.87
6	13.52	5.01	.491	27.54
7	12.06	4.37	.512	23.55
8	10.99	3.78	.492	22.34
9	9.86	3.20	.464	21.25
10	8.72	2.66	.425	20.52

REFERENCES

1. "Cerenkov Radiation from Bunched Electron Beams", F. R. Buskirk and J. R. Neighbours, Naval Posgraduate School School Report Number NPS-61-83-003 (October 1982)

2. "Cerenkov Radiation from Periodic Electron Bunches", F. R. Buskirk and J. R. Neighbours, to be published in Physical Review A.

3. Bazzlev, V. A., V. I. Glebov, E. I. Denisov, N. K. Zhevago, M. A. Kumakhov, A. S. Khlebnikov and V. G. Tsinoev. Sov. Phys. JETP 54(5), 884 (1981).

ACKNOWLEDGEMENT

This work was partially supported by the Naval Postgraduate School Foundation Research Program.

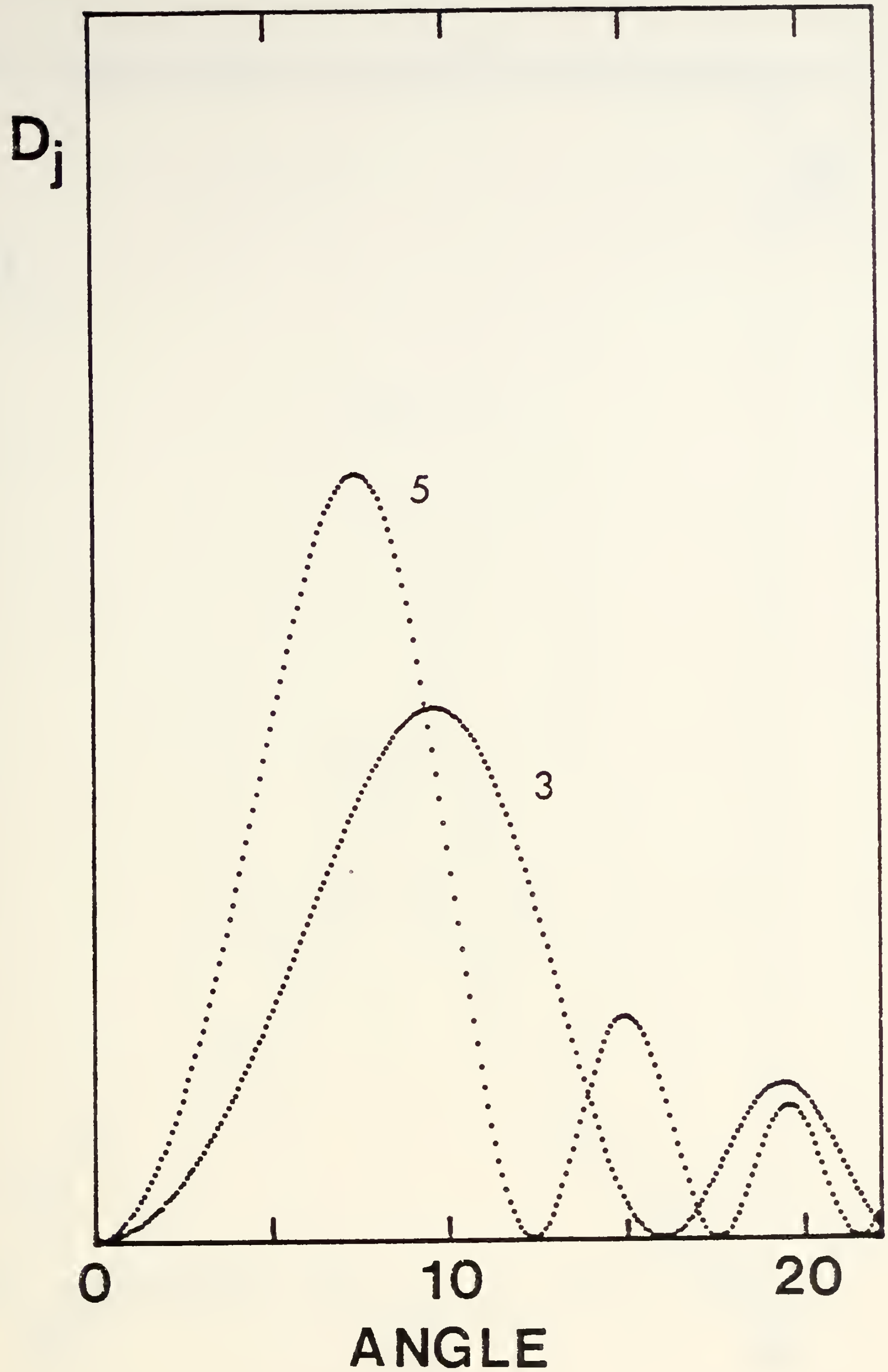

Figure 1. Radiation function D_j for the harmonics j = 3 and j = 5 as a function of emission angle. Vertical scale is arbitrary. For j = 3 the first maximum occurs at 9.8° and the first minimum occurs at 16.2°. For the 5th harmonic the values are 7.6° and 12.6° respectively. Fundamental frequency is 2.85 GH_z. Electron beam parameters and index of refraction are as described in the text. The emitted power per unit solid angle is equal to a constant multiplying Dj as stated in (8).

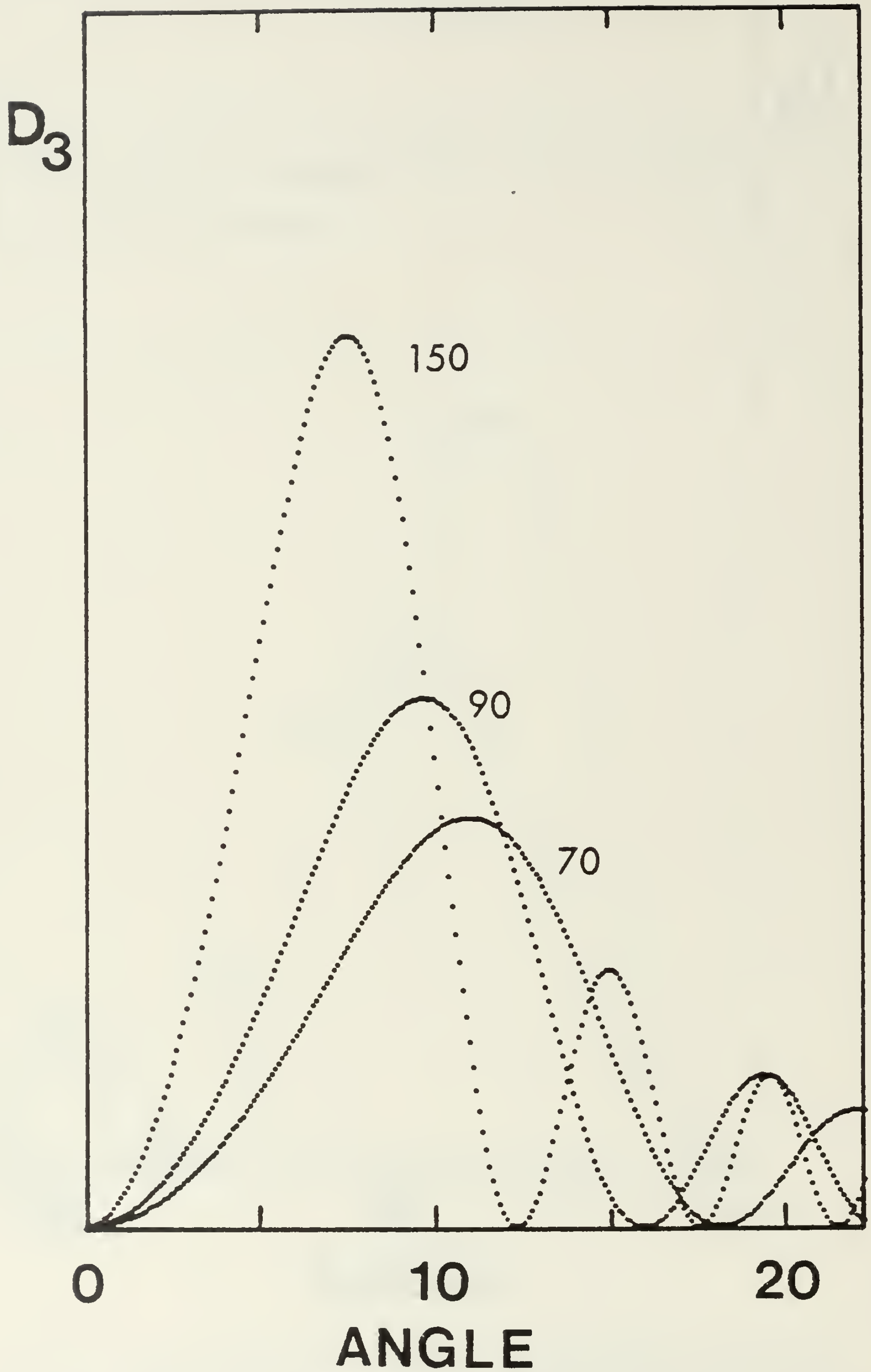

Figure 2. Third harmonic radiation function $D_3(\theta)$ as a function of emission angle for gas (air) cell lengths of 70, 90 and 150 cm. Vertical scale is arbitrary, and the same for all three curves. The peaks occurs at 11.1 9.8 and 7.6 degrees respectively. Electron beam parameters as given in the text.

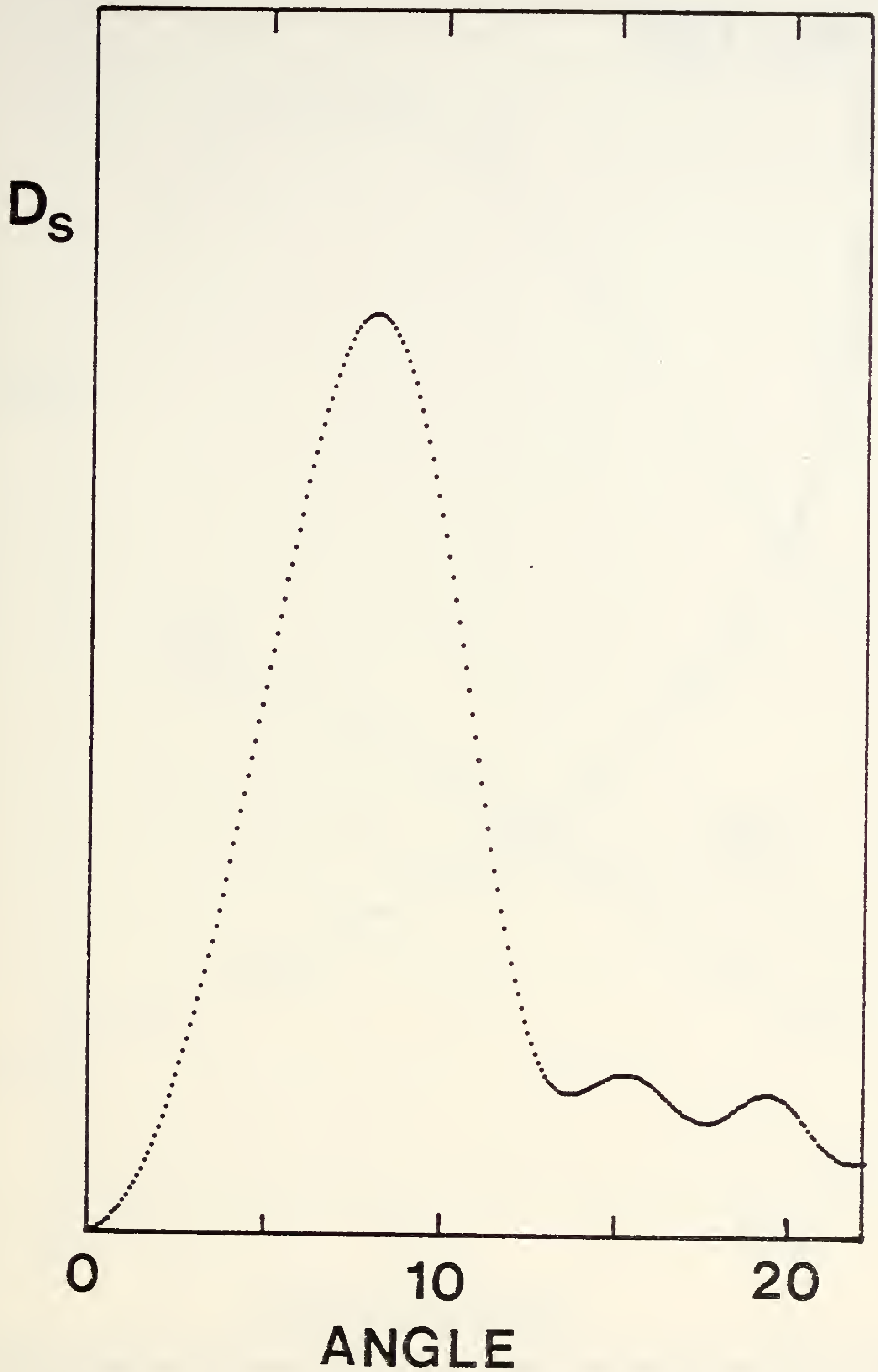

Figure 3. D_s, the sum of $D_j(\Theta)$ for j = 3, 4, 5 as a function of Θ. Electron beam parameters and index of refraction are as described in the text. The emitted power per unit slid angle is equal to a constant times D_s. The maximum in D_s occurs at Θ = 8.30°.

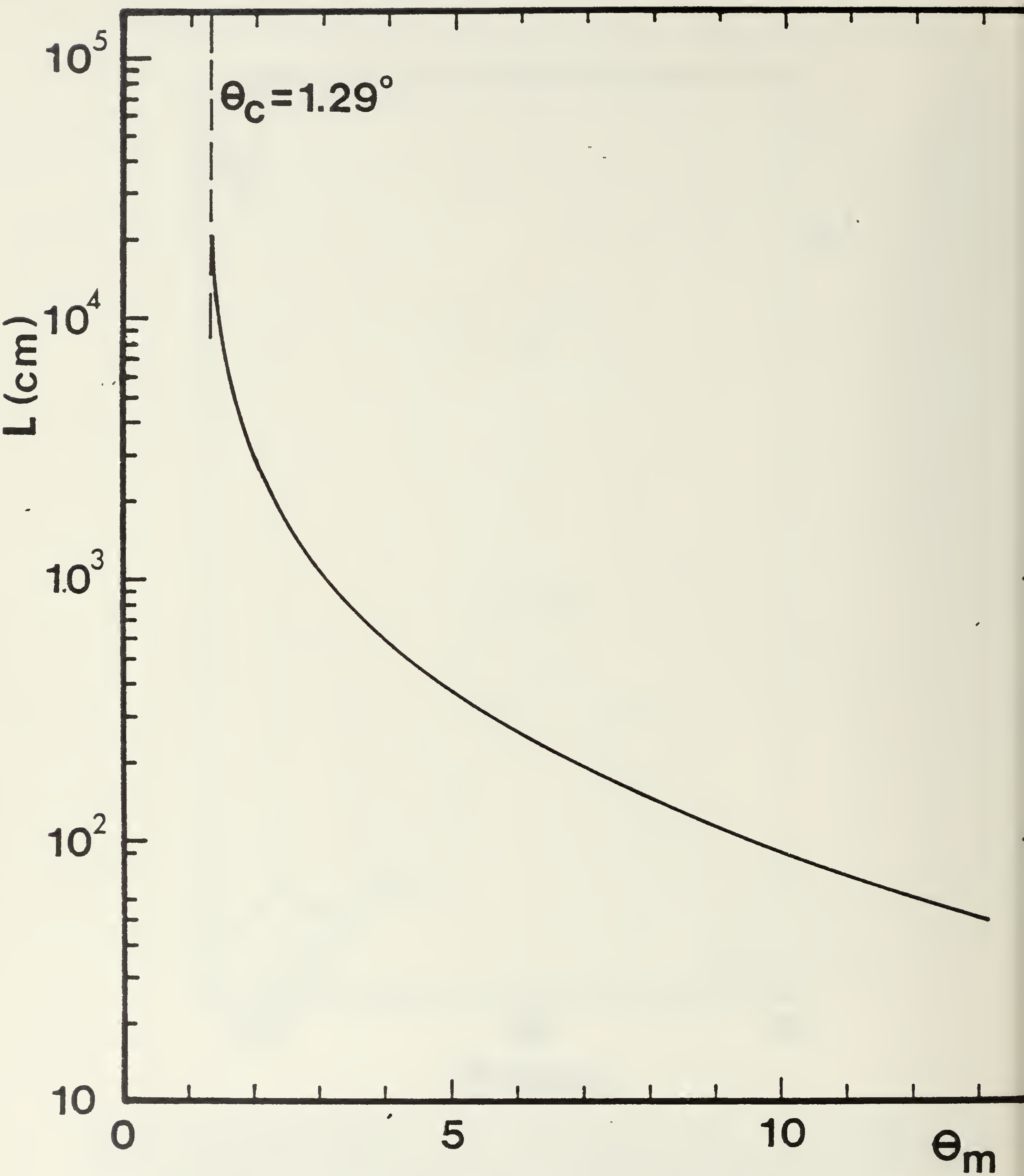

Figure 4. Dependance of the first maximum in $D_3(\theta)$ as a function of gas (air) cell length.

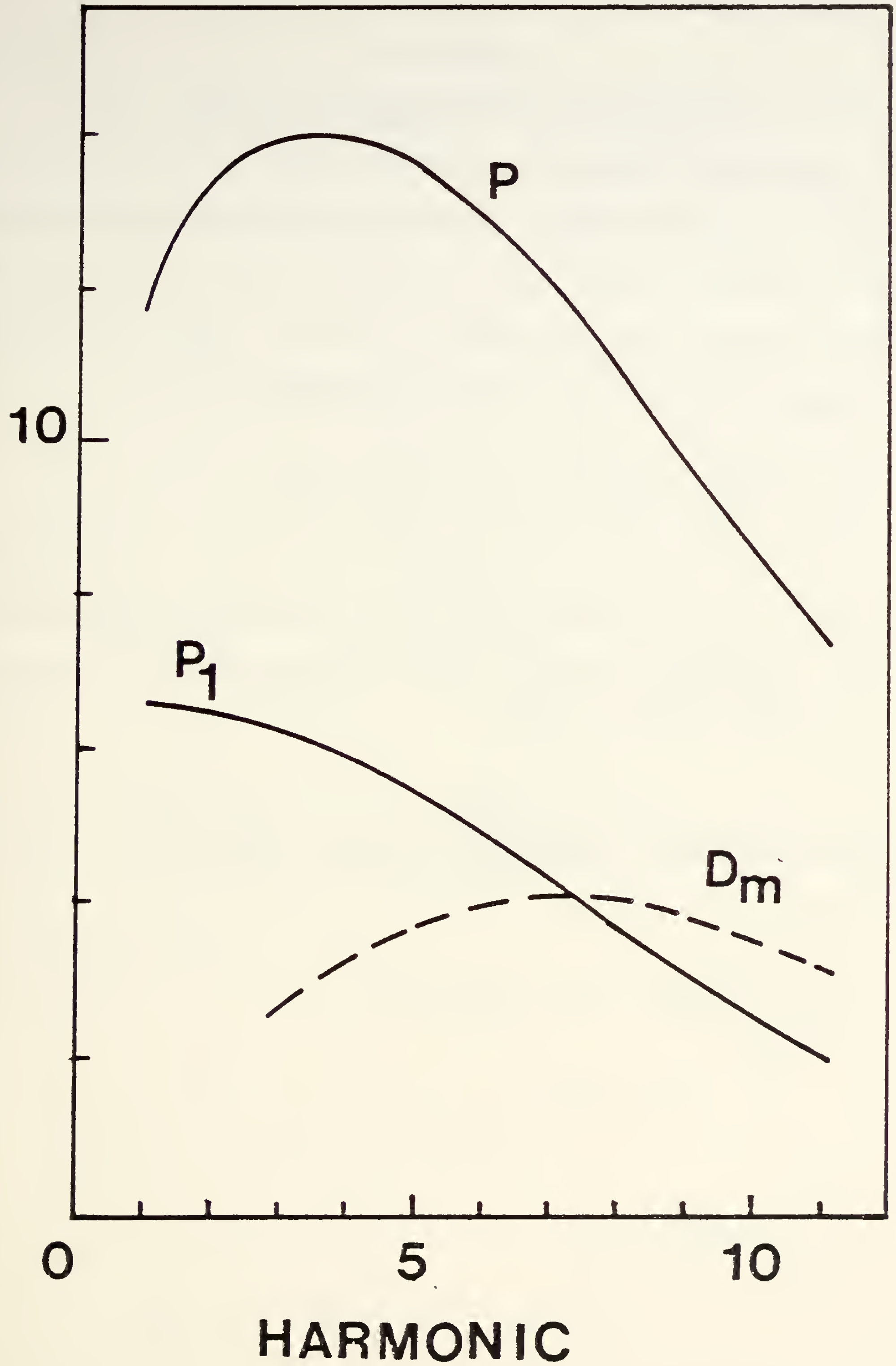

FIGURE 5. Radiated power as a function of linac harmonic number. P is the total radiated power obtained by integrating D_j from 0 to π. P_1 is the radiated power in the first lobe of the radiation pattern. D_m is the maximum value of D_j; i.e., the value of D_j at θ_m. All calculations are for L = 90 cm and other parameters as given in the text. The vertical scale is in cm^2. Actual powers can be obtained by multiplying by $2\pi Q$.

Appendix A

Discussion of the Power Increase.

An approximate expression for the power radiated is developed. It is the object of this section to explain the large increase in power found when the present results are compared to the results of the earlier paper. We start from the expression for the power radiated per unit solid angle, .

$$W = \frac{2\mu\nu_o^4 q^2}{c} \frac{j^2L^2}{4} \sin^2\theta \frac{\sin^2 u}{u^2} F^2(\vec{k}) \tag{A1}$$

where

$$u = \frac{kL}{2} (\cos\theta_c - \cos\theta) \tag{A2}$$

The total power radiated at a given frequency ν, where $\nu = j\nu_o$, is

$$P_\omega = \iint W d\Omega = 2\pi \int W d(\cos\theta) \tag{A3}$$

$$P_\omega = A_2 I$$

where

$$A_2 = \frac{4\mu\nu_o^4 q^2}{c} \frac{2\pi}{kL} \tag{A4}$$

and

$$I = \frac{j^2L^2}{4} \int \mathrm{Sin}^2\theta \frac{\sin^2 u}{u^2} F^2(\vec{k})\, du \tag{A5}$$

Approximation 1.

If we assume $\sin^2\theta$ is constant and equal to $\sin^2\theta_c$ and assume $F^2(\vec{k})$ is constant, the integral of $\sin^2 u/u^2$ gives π. Then the approximation to the integral, denoted by I_1, is given by

$$I_1 = \frac{j^2L^2}{4} F^2(\vec{k}) \sin^2\theta_c \cdot \pi \tag{A6}$$

Approximation 2.

A better approximation to A5 is obtained by still considering $F^2(\vec{k})$ constant, but considering $\sin^2\theta$ to be a variable. Then we have for I

$$I = \frac{j^2L^2}{4} F^2(\vec{k}) \int \sin^2\theta \frac{\sin^2 u}{u^2} du \tag{A7}$$

where u is given by A2.

Now note that

$$du = -\frac{kL}{2} d(\cos\theta) = \frac{kL}{2} \sin\theta \, d\theta \tag{A8}$$

Thus

$$I = \frac{j^2L^2}{4} F^2(\vec{k}) \frac{kL}{2} \int \sin^3\theta \frac{\sin^2 u}{u^2} d\theta \tag{A9}$$

When the integrand is plotted, it may be noted that the function to be integrated has a peak at θ_m equal to about half the value of θ_n, where θ_n is the value of θ at the first null beyond u = o. Furthermore $\sin^2 u/u^2$ is found to be near unity at θ_m.

We thus arrive at a slightly better approximation to I, denoted by I_2, by letting $\sin\theta = \theta_n/2$, and the width of the peak be $\theta_n/2$.

$$I_2 = \frac{j^2 L^2}{4} F^2(\vec{k}) \frac{kL}{2} \left(\frac{\theta_n}{4}\right)^4 \tag{A10}$$

Then the ratio of I_2/I_1 is

$$\frac{I_2}{I_1} = \frac{kL}{2\pi} \left(\frac{\theta_n}{4}\right)^4 \frac{1}{\sin^2\theta_c} \tag{A11}$$

If we let $\theta_c = 1.29°$, k correspond to the 3^{rd} harmonic of 2.85 GH_z, and L = 90 cm, we find $\theta_n = 16.1°$, and

$$\frac{I_2}{I_1} = 19.7 \tag{A12}$$

This value corresponds to the value 19.6 obtained by doing the integral. The significant result is that the large increase in power is associated with the broad diffraction lobe of this physical situation which couples with the $\sin^2\theta$ dependence of the Cerenkov power formula. Thus when the peak is shifted from 1.29° expected for an infinite cell, large increases in power are expected.

Appendix B

Angles for Minima in the Diffracted Cerenkov Radiation

The radiation functon $Dj(\theta)$ given by (9) and therefore the radiated power (8) go to zero when the diffraction variable u takes on successive values which are integer nultiples of π. From (10) this condition is

$$kL\ (\cos\theta_c - \cos\theta) = n\ 2\pi \qquad \text{(B1)}$$

For a given frequency and cell length, B1 can be solved for the values of the angle which give a minimum in $\dot{W}$.

$$\cos\theta = \cos\theta_c - n\,\frac{2\pi}{kL} \qquad \text{(B2)}$$

Or in terms of the fundamental frequency and the harmonic number of the Cerenkov radiation, the result is

$$\cos\theta = \cos\theta_c - \frac{n}{j}\,\frac{c}{\nu_o L} \qquad \text{(B3)}$$

Figure B-1 is a plot of the first four minimum angles for harmonics up to j = 20. Other parameters are as discussed previously. The figure shows the dimunition of the diffraction lobes as j increases (kL increases).

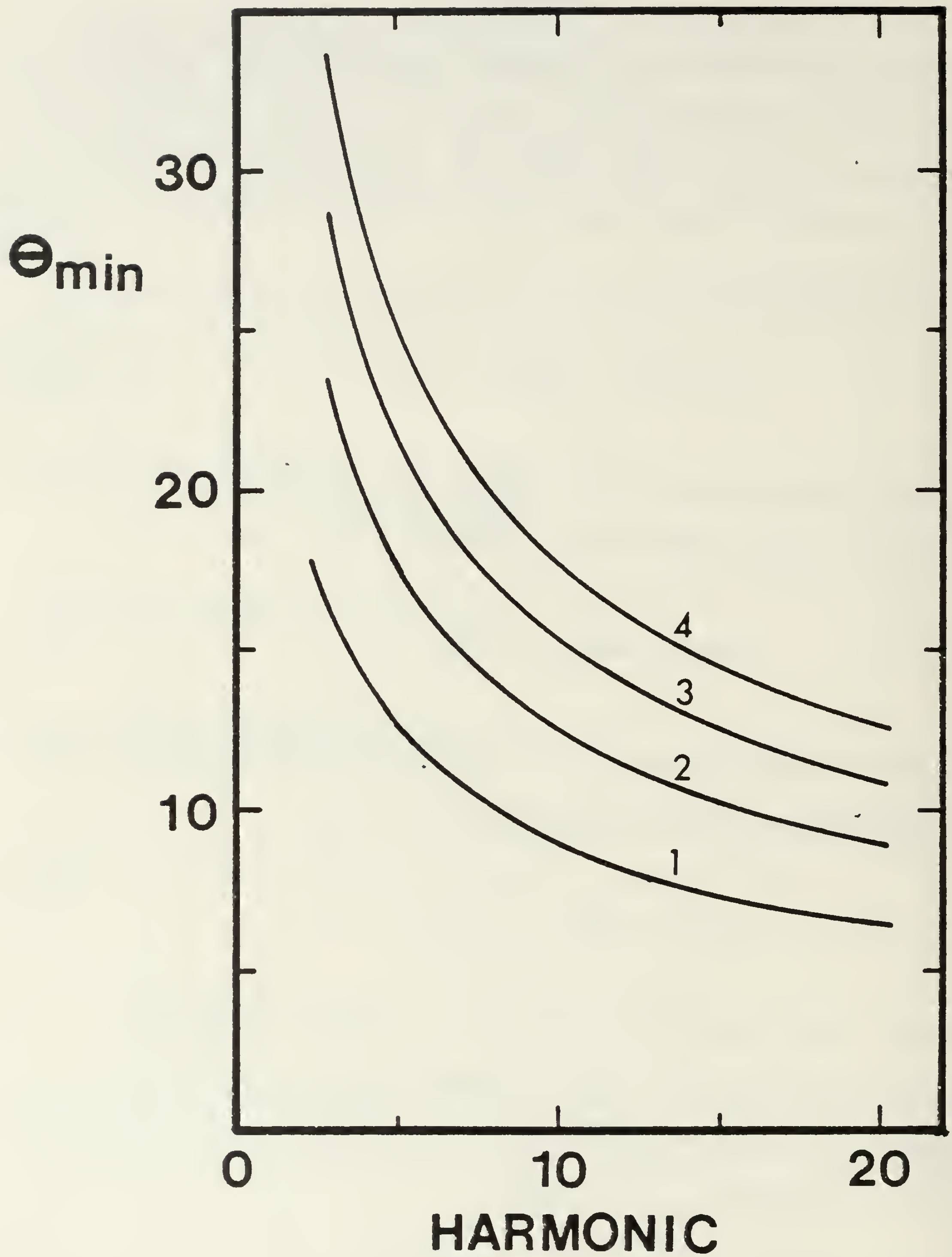

Figure B1. Shift in angles at which the diffracted Cerenkov radiation is zero as a function of harmonic number of the radiation. Cell length is assumed to be 90 cm and other parameters are as described in the text.

DISTRIBUTION LIST

Office of Naval Research CDR T. Swafford 800 N. Qunicy St. Arlington, VA. 22217	5
Office of Naval Research CDR James Offutt 1030 E. Green St. Pasadena, CA. 91106	2
Library, Naval Postgraduate School Office of Research Administration NPS, Code 012A Monterey, CA. 93940	4
F.R. Buskirk & J.R. Neighbours Naval Postgraduate School Physics Dept., Code 61 Monterey, CA. 93940	20
Dr. Rainer Pitthan Bin 96 SLAC Stanford, CA 94305	1
Dr. Thomas Starke M4, M.S. P942 Los Alamos National Laboratory Los Alamos, N.M. 87545	1
MAJ E. W. Pogue M4, M.S. P942 Los Alamos, National Laboratory Los Alamos, N.M. 87545	1
Dr. Charles Bowman D442 Los Alamos, National Laboratory Los Alamos, N.M. 97545	1
Dr. Thomas Fessenden L-436 Lawrence Livermore Laboratory Box 808 Livermore, CA 94550	1